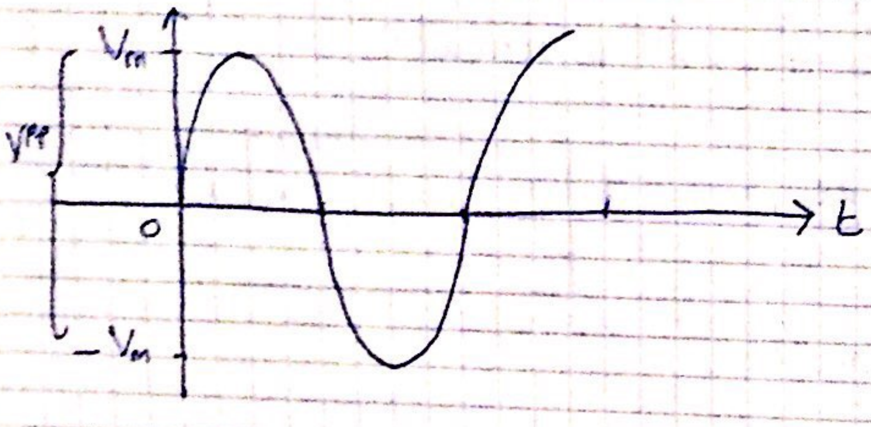
Electrocinétique des courants Alternatifs.

Le courant Alternatif :

$$V(t) = V_m \sin(\omega t) = V_m \sin(2\pi f t)$$

pulsation $\omega = 2\pi f = \dfrac{2\pi}{T}$



Valeur moyenne d'une grandeur périodique : (position continu).

$$X_{moyen} = \langle x \rangle = \frac{1}{T} \int_T x(t)\, dt$$

Valeur efficace d'une grandeur périodique : (position Alternatif.

$$X_{eff} = \sqrt{\langle x^2 \rangle} = \sqrt{\frac{1}{T} \int_T x^2(t)\, dt}$$

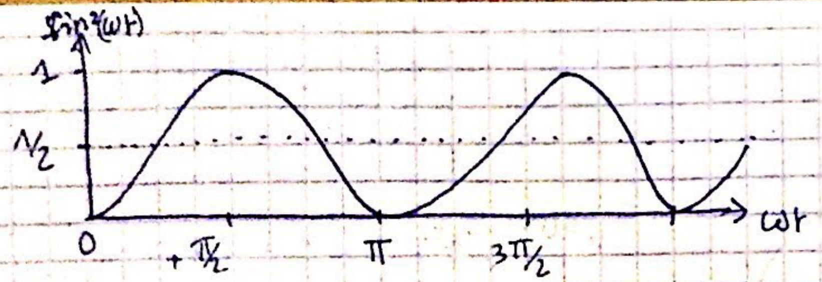
Tension et Intensité efficaces :

$$I_{eff} = \sqrt{\langle i^2 \rangle}$$

$$\langle i^2(t) \rangle = \langle I_m^2 \sin^2 \omega t \rangle = I_m^2 \langle \sin^2 \omega t \rangle$$

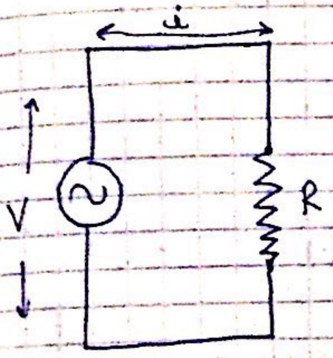
$$\langle \sin^2 \omega t \rangle = 1/2$$

Scanned with CamScanner



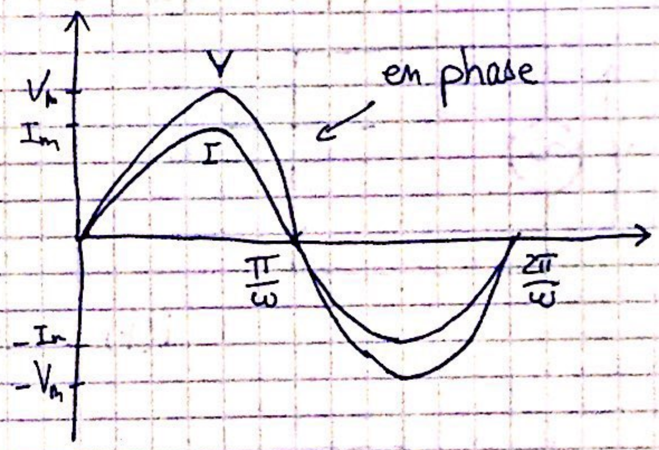
$$\Rightarrow \boxed{I_{eff} = \frac{I_m}{\sqrt{2}}} \qquad \boxed{V_{eff} = \frac{V_m}{\sqrt{2}}}$$

Résistances en courant Alternatif :



$$i(t) = \frac{V(t)}{R} = \frac{V_m}{R} \sin \omega t$$

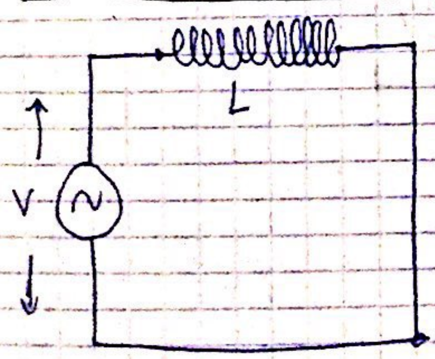
$$I_m = \frac{V_m}{R}$$



$$\Rightarrow \quad t = \frac{n}{2f} = \frac{n\pi}{\omega}$$

$$V(t) = V_m \sin(\omega t)$$

Inductances en courant Alternatif :



$$V(t) = L \frac{di}{dt}$$

$$\Rightarrow \quad V_m \sin(\omega t) = L \frac{di}{dt}$$

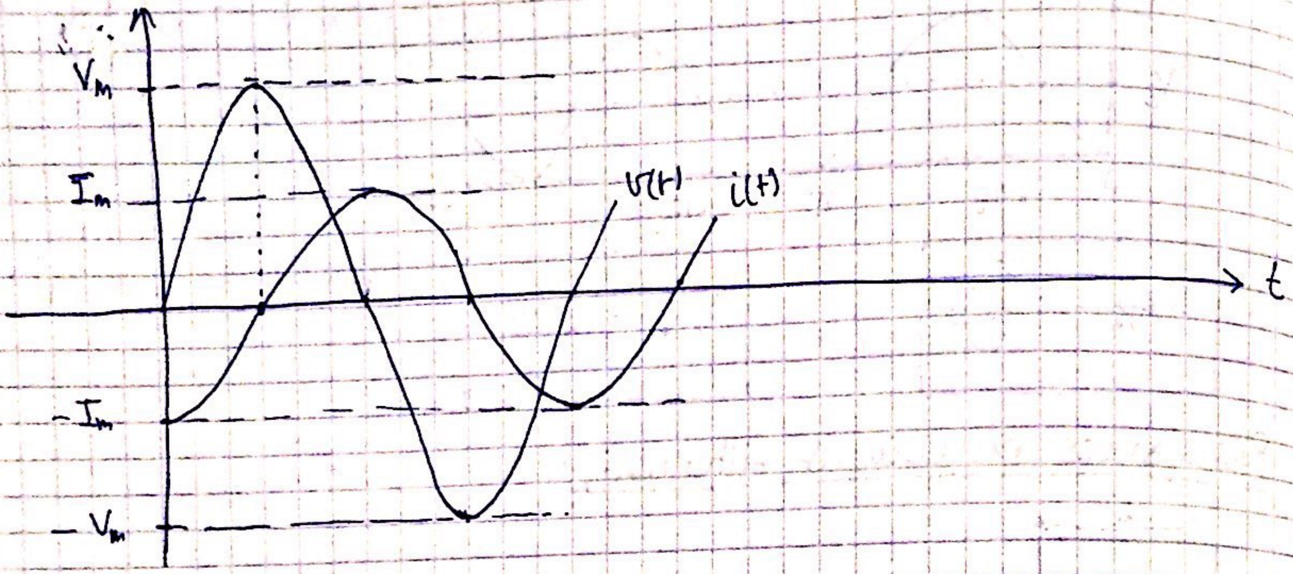
$$\int \frac{V_m}{L} \sin \omega t \, dt = \int di \Rightarrow \quad i(t) = -\frac{V_m}{\omega L} \cos \omega t$$

$$\Rightarrow \boxed{i(t) = \frac{V_m}{\omega L} \sin\left(\omega t - \pi/2\right)}$$ déphasage $\pi/2$.

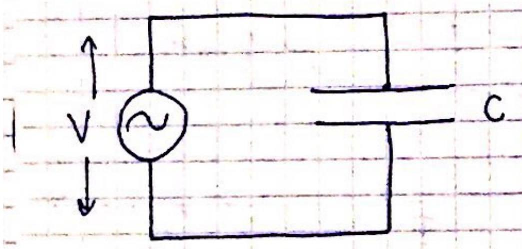
$$\varphi = -\frac{\pi}{2}$$

$V_{eff} = L\omega I_{eff}$ $\qquad X_L = L\omega$: Réactance inductive

$= 2\pi f L$



Condensateurs en courant Alternatif :



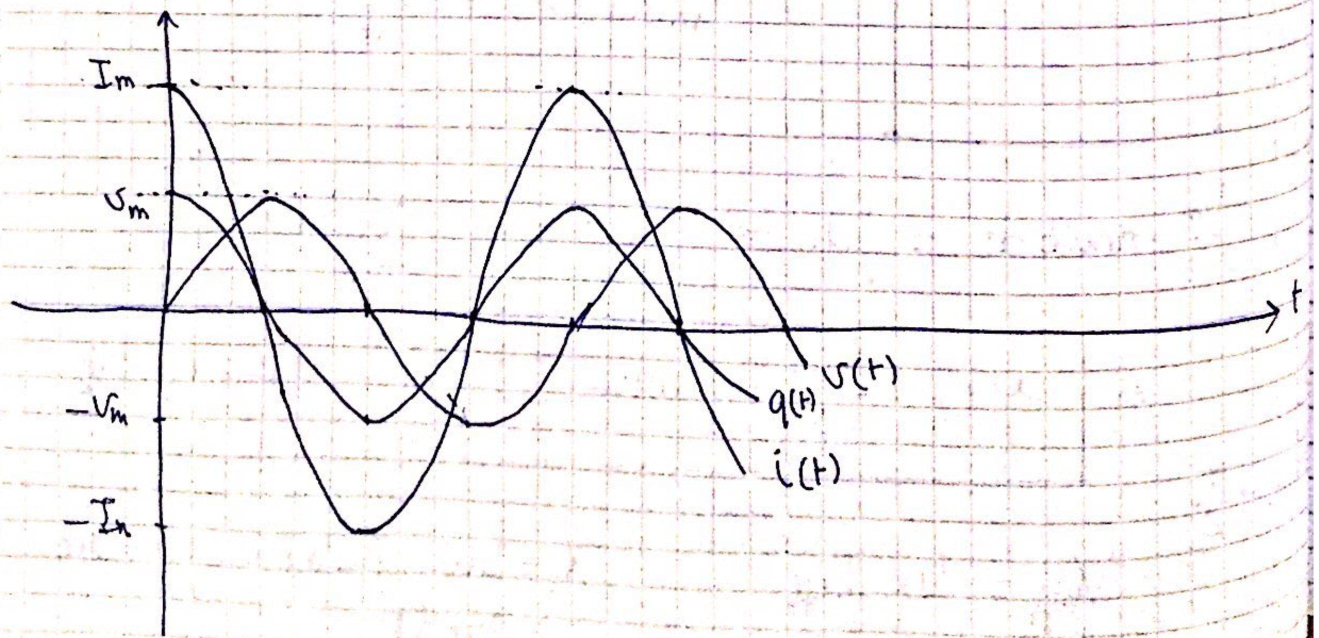
$$V(t) = V_m \sin(\omega t)$$

$$\Rightarrow q(t) = C V_m \sin \omega t$$

$$i(t) = C V_m \frac{d}{dt} \sin \omega t = C\omega V_m \cos \omega t$$

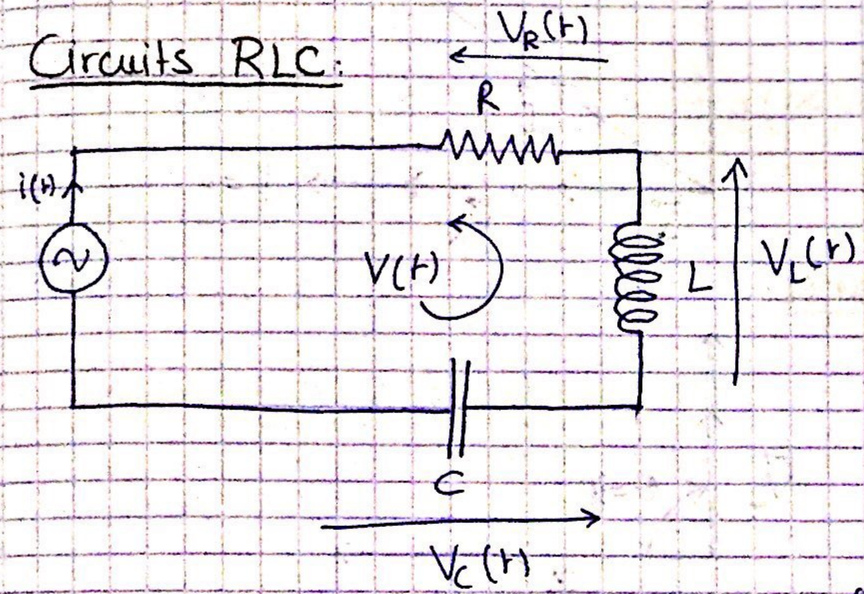
$$\Rightarrow \boxed{i(t) = C\omega V_m \sin\left(\omega t + \frac{\pi}{2}\right)}$$

$$\varphi = \frac{\pi}{2}$$



$$I_m = C\omega V_m$$

Circuits RLC :



$$\begin{cases} V_{Rm} = R I_m \\ V_{Lm} = L\omega I_m \\ V_{Cm} = \dfrac{I_m}{C\omega} \end{cases}$$

$$i(t) = I_m \sin \omega t$$

$$V_R(t) = R\,i(t) = R\, I_m \sin \omega t$$

$$V_L(t) = L\omega I_m \sin(\omega t + \pi/2)$$

$$V_C(t) = \frac{I_m}{C\omega} \sin(\omega t - \pi/2)$$

Représentation de Fresnel :

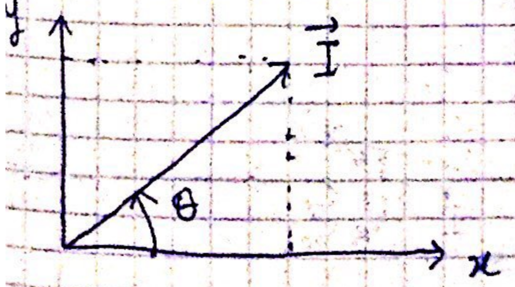
$$i(t) = I_m \sin(\omega t + \theta) = I_{eff}\sqrt{2}\,\sin(\omega t + \theta)$$

I_{eff} : intensité efficace. (I en A)

ω : pulsation (en rad/s)

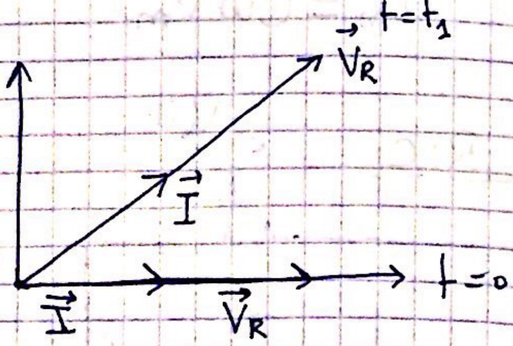
θ : phase (en rad)

On associe un vecteur $\vec{I}$, vecteur de Fresnel.



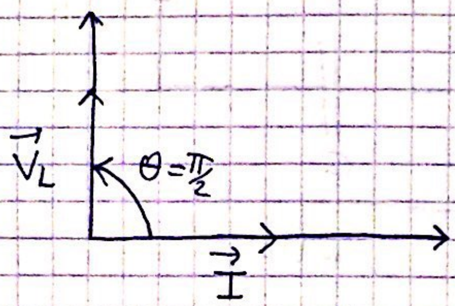
Pour une Résistance :

$\theta = 0$



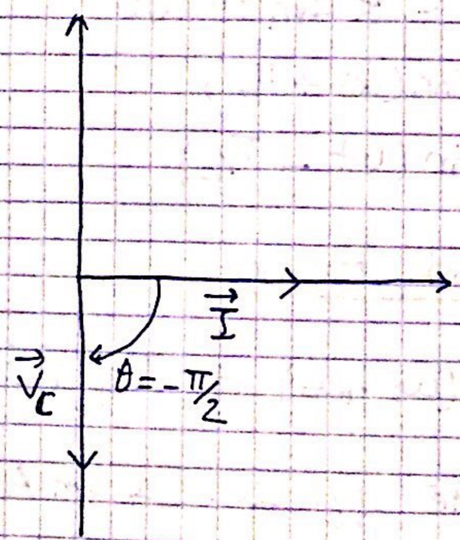
Pour une Inductance :

$\theta = \pi/2$



Pour une Capacité :

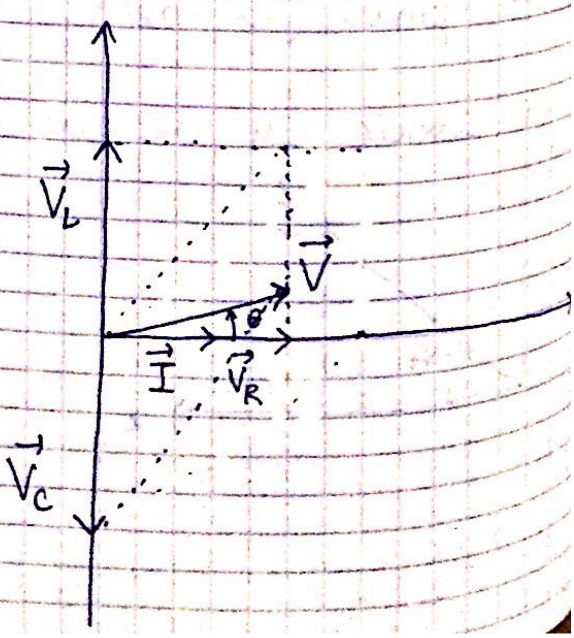
$\theta = -\pi/2$



Pour le Circuit RLC :

$$V(t) = V_R + V_C + V_L$$

$$\Rightarrow \quad \vec{V} = \vec{V}_R + \vec{V}_C + \vec{V}_L$$



$$\Rightarrow \quad V(t) = V_m \sin(\omega t + \theta)$$

$$\theta = \arctan\left(\frac{X_L - X_C}{R}\right) = Arg(\underline{Z})$$

$$\theta = \arctan\left(\frac{L\omega - \frac{1}{C\omega}}{R}\right) = Arg(\underline{Z}).$$

avec : $$|\underline{Z}| = \frac{V_m}{I_m} = \sqrt{R^2 + (X_L - X_C)^2}$$

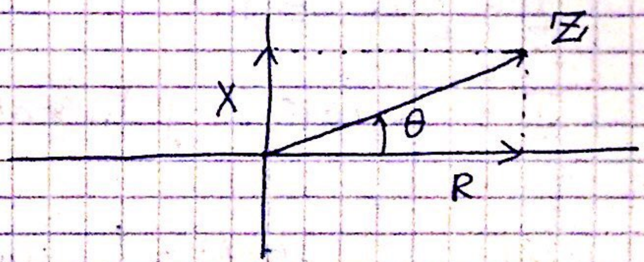
Impédance des circuits RLC :

$$V_{eff} = I_{eff}\sqrt{R^2 + (X_L - X_C)^2}$$

$$X = (X_L - X_C) \Rightarrow V_{eff} = I_{eff}\sqrt{R^2 + X^2}$$

L'Impédance : $$\|\underline{Z}\| = |\underline{Z}| = Z = \sqrt{R^2 + X^2}$$

$$\boxed{V_{eff} = I_{eff}\, Z}$$



$$\theta = \arccos(R/Z)$$

Impédance des Circuits à 2 éléments :

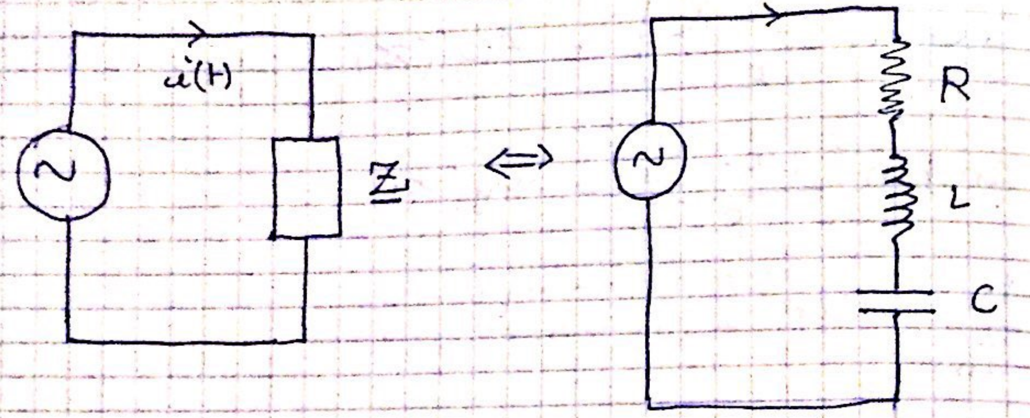
RC : $$Z = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + \left(\frac{1}{C\omega}\right)^2}$$

RL : $$Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + (L\omega)^2}$$

NB : les Inductances se combinent comme des résistances :

- en parallèle : $$\frac{1}{L_{eq}} = \sum_i \frac{1}{L_i}$$

- en série : $$L_{eq} = \sum_i L_i$$



$$\underline{Z} = \underline{Z}_R + \underline{Z}_C + \underline{Z}_L$$

$\underline{Z}_R = R = Z_R$

Représentation Complexe :

Nous pouvons aussi associer à $i(t)$ un nombre complexe.

$$\underline{I} = [I_m , \theta]$$

ou $$\underline{I} = I_m \cos\theta + j\, I_m \sin\theta .$$

pour $u(t)$: $U(t) = U_m \sin(\omega t) \Rightarrow \underline{U} = [U_m , 0]$

Exemple :

$$i = 2\sqrt{2} \sin\left(314t + \frac{\pi}{6}\right)$$

et $u = 220 \sin(314t)$

Valeurs réelles :

$\omega = 314$ rad/s

$\theta = \frac{\pi}{6}$

$U = 220V$

$I = 2\sqrt{2}$ A

valeurs complexes :

$$\underline{I} = 2\sqrt{2}\left(\cos\frac{\pi}{6} + j\sin\frac{\pi}{6}\right) = \sqrt{6} + j\sqrt{2}$$

$$\underline{U} = 220\left(\cos 0 + j\sin 0\right) = 220$$

Impédance complexe d'un dipole. $\boxed{\underline{Z} = \frac{\underline{U}}{\underline{I}}}$

$\underline{U} = 220$ et $\underline{I} = \sqrt{6} + j\sqrt{2}$

$$\underline{Z} = \frac{\underline{U}}{\underline{I}} = \frac{220}{\sqrt{6}+j\sqrt{2}} = 67{,}4 + j\,38{,}9$$

Résistance pure : R

$\underline{Z} = R \quad , \quad \theta = 0$

$\underline{Z} = \frac{\underline{U_R}}{\underline{I}}$

$\underline{U_R} = R\,I_m$

Inductance pure : L

$\underline{Z} = jL\omega \quad , \quad \theta = \frac{\pi}{2}$

$\underline{U_L} = jL\omega\, I_m$

$\underline{Z} = jL\omega = \frac{\underline{U_L}}{\underline{I}}$

Condensateur : C

$\underline{Z} = -j\frac{1}{C\omega} = \frac{1}{jC\omega} \quad , \quad \theta = -\frac{\pi}{2}$

$\underline{U_C} = -j/_{C\omega}\, I_m$

$\Rightarrow \underline{Z} = -j/_{C\omega}$

Exercice 3 : (TD N°5)

1) la pulsation :

$\omega = 2\pi f = 2\pi \times 50 = 314$ rad/s

2) $\underline{Z}_R = R = 30\,\Omega$

3) $\underline{Z}_L = jL\omega = j \times 0{,}2 \times 314 = 62{,}8j$

4) $\underline{Z}_C = \frac{-j}{C\omega} = \frac{-j}{100 \times 10^{-6} \times 314} = -31{,}8\,j$

5) En série:

$\underline{Z} = \underline{Z}_R + \underline{Z}_L + \underline{Z}_C = 30 + 31\,j$.

Puissance fournie à un circuit RLC:

La puissance instantanée:

$$p(t) = v(t) \times i(t).$$

En complexe: $\boxed{\underline{P} = \frac{1}{2}\, \underline{V} \times \overline{\underline{I}}}$

puissance moyenne:

Dans une résistance:

$$p = R\, i^2(t) = R\, I_m^{\,2} \sin^2(\omega t)$$

$$\Rightarrow \langle p \rangle = R I_{eff}^2 = P \quad \text{et} \quad Q = 0$$

Dans un condensateur:

$$p = U_m \sin\left(\omega t - \frac{\pi}{2}\right) I_m \sin(\omega t) = -\frac{U_m I_m}{2} \sin(2\omega t)$$

$$\Rightarrow \langle p \rangle = 0 \Rightarrow P = 0 \quad \text{et} \quad Q = \frac{-1}{C\omega} I_{eff}^2$$

Dans une inductance:

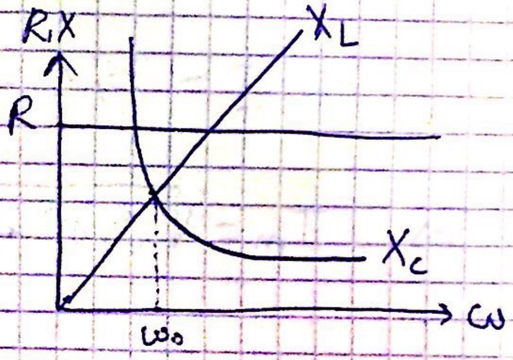
$$p = U_m \sin\left(\omega t + \frac{\pi}{2}\right) I_m \sin(\omega t) = \frac{U_m I_m}{2} \sin(2\omega t)$$

$$\Rightarrow \langle p \rangle = 0 \Rightarrow P = 0 \quad \text{et} \quad Q = L\omega I_{eff}^2$$

$\underline{P} = P + jQ$, $P = \langle p \rangle$: puissance active.
Q : puissance réactive.

$\Rightarrow \langle p \rangle = V_{eff} I_{eff} \cos\theta$ et $Q = V_{eff} I_{eff} \sin\theta$

Résonance dans un circuit RLC :



$$Z = \sqrt{R^2 + (X_L - X_c)^2}$$

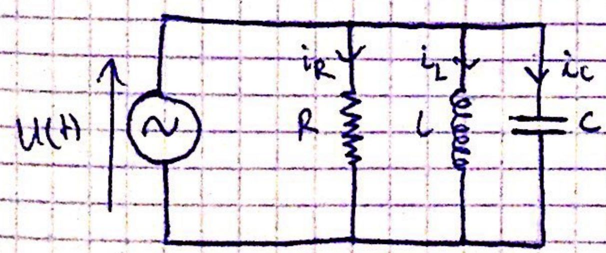
$\Rightarrow \omega_0 = 2\pi f_0$

$$X_L = X_c \Rightarrow L\omega_0 = \frac{1}{C\omega_0} \Rightarrow \omega_0^2 = \frac{1}{LC}$$

$$\Rightarrow f_0 = \frac{1}{2\pi\sqrt{LC}}$$

La résonnance permet de trouver la fréquence.

Ex 1 : (TD N°5 :)



1) $u(t) = U_m \sin(\omega t)$ $\underline{U} = U_m = 1V$

$$\underline{I}_R = \frac{\underline{U}}{R} , \quad \underline{I}_L = \frac{\underline{U}}{jL\omega} , \quad \underline{I}_c = \frac{\underline{U}}{\frac{1}{jC\omega}} = jC\omega\,\underline{U}$$

$$\Rightarrow \underline{I}_R = \frac{1}{10} \Rightarrow i_R(t) = 0{,}1 \sin(1000t)$$

$$\Rightarrow \underline{I}_L = \frac{1}{j \times 10} = -j\,0{,}1 \Rightarrow i_L(t) = 0{,}1 \sin\left(1000t - \frac{\pi}{2}\right)$$

$\Rightarrow \underline{I}_c = 1 \times j \times 0{,}05 \Rightarrow i_c(t) = 0{,}05 \sin\left(1000t + \frac{\pi}{2}\right)$

$i(t) = i_R(t) + i_L(t) + i_c(t)$

$\Rightarrow \underline{I} = \underline{I}_R + \underline{I}_c + \underline{I}_L$

$\Rightarrow \underline{I} = 0{,}1 + (-0{,}1)\,j + j\,(0{,}05)$

$\Rightarrow \underline{I} = 0{,}1 - 0{,}05\,j$

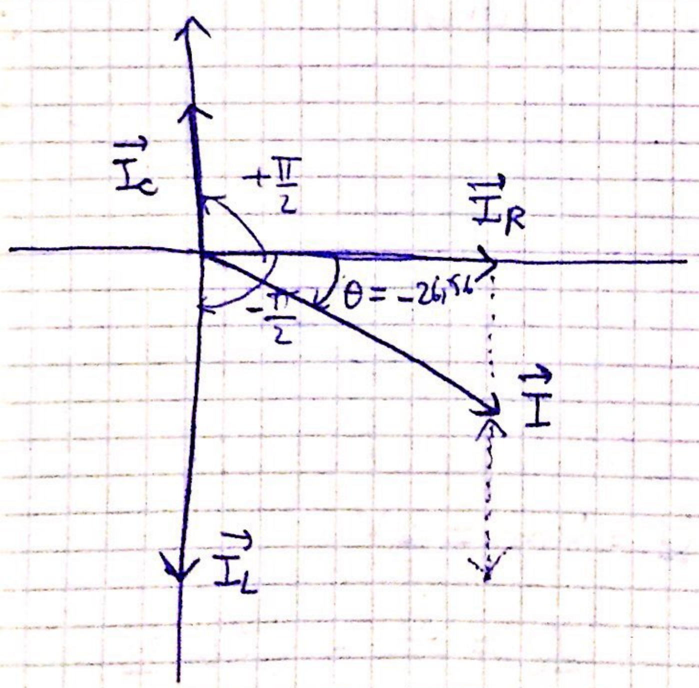
$\Rightarrow \quad I_m = \sqrt{(0{,}1)^2 + (0{,}05)^2}$ et $\theta = \arctan\left(\frac{-0{,}05}{0{,}1}\right) = -26{,}56°$

$i(t) = I_m \sin(\omega t + \theta) = 0{,}112 \sin(1000t - 26{,}56°)$

2)

$\underline{I}_R = [0{,}1\,,\,0]\;,\; \underline{I}_L = [0{,}1\,,\,-\frac{\pi}{2}]$

$\underline{I}_C = [0{,}05\,,\,+\frac{\pi}{2}]$



3) La Résonnance $\Rightarrow I_{L_m} = I_{C_m} \Rightarrow$ Il reste uniquement $\vec{I_R}$